8. R. White, "Laminar separation and reattachment behind a downstream-facing step at hypersonic Mach number, including the effect of the approach boundary layer," Medd. Flygtekn. Försöksanst, No. 103 (1956).
9. D. Chapman, D. Kuehn, and H. Larson, "Investigation of separated flow in supersonic and subsonic streams with emphasis on the effect of transition," NACA Rep. No. 1356 (1958).
10. Zakkay and Cresci, "Experimental investigation of the near wake behind a slender cone at M = 8 and 12," Raketn. Tekh. Kosm., No. 1 (1966).
11. L. Lokman, Measurement of the base pressure and the heating in flow over blunt and sharp cones in a shock tube, Raketn. Tekh. Kosm., No. 10 (1967).
12. D. Chapman, "Analysis of base pressure at supersonic velocities and comparison with experiment," NACA Rep. No. 1051 (1951).
13. J. E. Nash, "An analysis of two-dimensional turbulent base flow, including the effect of the approaching boundary layer," ARC RM No. 3344 (1963).
14. A. N. Antonov, "Calculation of the interaction of a turbulent boundary layer with the external supersonic flow behind an obstacle," Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza, No. 3 (1971).
15. A. N. Antonov, "Calculation of the interaction of a turbulent boundary layer with the external supersonic flow in a concave corner and on the spherical base part of the body," Zh. Prikl. Mekh. Tekh. Fiz., No. 1 (1976).
16. S. Donaldson, "The effect of sting supports on the base pressure on a blunt-base body in a supersonic stream," Aeron. Quart., 6, No. 2 (1955).

SOLITONS IN A DOWN-FLOWING FILM WITH MODERATE MASS FLOW RATES OF THE LIQUID

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UDC 532.51

Using the hypothesis of self-similarity, in [1] an equation was obtained describing long-wave perturbations in a vertical film of liquid with moderate mass flow rates:

$$\left(\frac{\partial}{\partial t}+3\frac{\partial}{\partial x}\right)h+6h\frac{\partial h}{\partial x}-\frac{2}{15}\mathrm{Re}\frac{\partial}{\partial t}\left(h\frac{\partial h}{\partial t}\right)+\frac{\mathrm{Re}}{3}\left(\frac{\partial}{\partial t}+1.69\frac{\partial}{\partial x}\right)\left(\frac{\partial}{\partial t}+0.71\frac{\partial}{\partial x}\right)h+\mathrm{W}\frac{\partial^4 h}{\partial x^4}=0, \qquad (1)$$

where $\mathrm{Re} = gh_0^3/3\nu^2$; $\mathrm{W} = \sigma/\rho h_0^2$; h is the shift of the surface of the film from the unperturbed level, measured in units of h_0; and h_0 is the thickness of the unperturbed film.

For a steady-state running wave $h = h(x - ct)$, from (1) we obtain

$$(3-c)h' + 6hh' - 2\,\mathrm{Re}\,c^2(hh')'/15 + \mathrm{Re}(1.69-c)(0.71-c)h''/3 + \mathrm{W}h^{\mathrm{IV}} = 0 \qquad (2)$$

(a prime means differentiation with respect to x).

In finding soliton solutions of Eq. (2), it can be integrated once:

$$(3-c)h + 3h^2 - 2\,\mathrm{Re}\,c^2hh'/15 + \mathrm{Re}\,(1.69-c)(0.71-c)h'/3 + \mathrm{W}h''' = 0. \qquad (3)$$

Using the replacement

$$\begin{gathered} h = aH,\ x_1 = bx, \\ a = \mathrm{W}b^3,\ b = (\mathrm{Re}(1.69-c)(0.71-c)/3\mathrm{W})^{1/2} \end{gathered} \qquad (4)$$

Eq. (3) is brought to the form

$$-c_1H + 3H^2 - 2mHH' - H' + H''' = 0, \qquad (5)$$

where

$$\begin{gathered} c_1 = (c-3)(3/(z(1.69-c)(0,71-c)))^{3/2}, \\ m = c^2z((1.69-c)(0.71-c)/3)^{1/2}/15,\ z = (\mathrm{Re}^3/\mathrm{W})^{1/2}. \end{gathered} \qquad (6)$$

Relationships (4)-(6) are valid if

$$c > 1.69 \quad \text{or} \quad c < 0.71.$$

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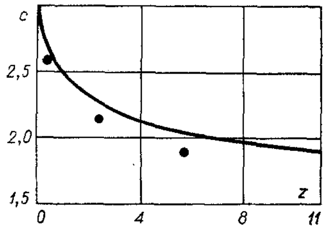

Fig. 1

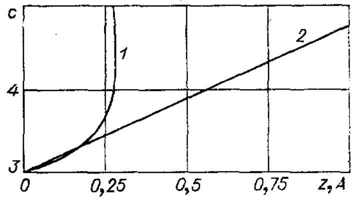

Fig. 2

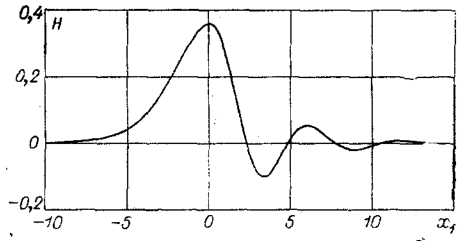

Fig. 3

Thus, the problem is brought down to the solution of Eq. (5); for a given value of m, the eigenvalue c_1 and the soliton function H are found. The values of the velocities of the solitons c and the corresponding values of z(c) are found from (6), and of the function of h, from (4).

Representing the soliton solution of Eq. (5) in the form of a Fourier integral

$$H = \int H_k \exp(ikx_1)\, dk,$$

for determination of H_k we obtain the integral equation

$$H_k = (3 - ikm) \int H_{k'} H_{k-k'} dk' / (c - i(k - k^3)). \quad (7)$$

By virtue of the invariance of Eq. (5) with respect to the replacement

$$c_1 \to -c_1,\ H \to -H,\ x_1 \to -x_1,\ m \to -m$$

it is sufficient to consider only the region of values of the parameter $m \geqslant 0$. Equation (7) was solved by a method described in [2].

Depending on whether the value of $\int_{-\infty}^{\infty} h dx$ is greater or less than zero, the value of a soliton is relatively positive or negative.

Figure 1 gives the dependence of the velocity of negative solitons on the parameter z. For purposes of comparison, the points plot the data of [3], in which soliton solutions close to Eq. (2) are sought.

Figure 2 gives values of the velocities of the positive solitons found, as a function of z (curve 1). Such solitons can exist only with values of $z \leqslant z_* = 0.2810$. Curve 2 gives the dependence of the amplitudes of these solitons on their velocity. Here $A \equiv (H_{max} - H_{min})a$. For $z = z_*$ the form of a soliton is shown in Fig. 3. Its velocity c = 4.405, and its amplitude A = 0.784.

For the case $z \ll 1$, soliton solutions were found in [2]. With finite values of the parameter z, positive solitons have not been observed earlier, although they are of great interest, since, in an experiment, wavy conditions are attained in the form of a sequence of positive solitons [4]. The presence of z_* makes it easier to understand why the thickness of the residual layer, over which these solitons are propagated, depends only slightly on the mass flow rate of the liquid Q. If the value of Q is such that $z > z_*$, the flow is restructured in such a way that the value of the parameter z, calculated from the thickness of the residual layer, is of the order of z_*, and the residual mass flow rate goes over into solitons. Thus, knowing z_* and Q, the number of solitons arising and the fraction of sections of the film with a flat boundary can be evaluated.

Such an evaluation can be useful for a number of problems of heat and mass transfer through the surface of a film.

LITERATURE CITED

1. S. V. Alekseenko, V. E. Nakoryakov, and B. G. Pokusaev, "Wave formation with the flow of a liquid film at a vertical wall," Zh. Prikl. Mekh. Tekh. Fiz., No. 6 (1979).
2. O. Yu. Tsvelodub, "Investigation of the hydrodynamic stability of flows in two-phase media," Author's Abstract of Candidate's Dissertation, Inst. Tekh. Fiz., Sib. Otd., Akad. Nauk SSSR, Novosibirsk (1978).

3. V. Ya. Shkadov, "Single waves in a layer of viscous liquid," Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza, No. 1 (1977).
4. S. V. Alekseenko, V. E. Nakoryakov, and B. G. Pokusaev, "Waves at the surface of a vertically down-flowing film of liquid," Preprint Inst. Tekh. Fiz., Sib. Otd., Akad. Nauk SSSR, No. 36-79, Novosibirsk (1979).

UNSTEADY ROTATION OF A CYLINDER IN A VISCOUS FLUID

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The flow of a viscous fluid around a cylinder set in rotational motion at constant angular velocity was investigated in [1, 2]. The present paper deals with the problem of rotation, in a viscous incompressible fluid, of a round cylinder, on unit length of which, beginning at time t = 0, there acts a constant moment of external forces M. The fluid flow is assumed to be plane. At $t \leqslant 0$ the cylinder and fluid are at rest.

We select cylindrical coordinates r, θ, and z in such a way that the z axis is directed along the cylinder axis. We assume that the flow velocity V is independent of θ. Then, as is easily verified, the r component of the vector V is zero, and the considered problem reduces to solution of the equations

$$\frac{\partial V_\theta}{\partial t} = \nu\left(\frac{\partial^2 V_\theta}{\partial r^2} + \frac{1}{r}\frac{\partial V_\theta}{\partial r} - \frac{V_\theta}{r^2}\right); \tag{1}$$

$$I\, d\Omega/dt = M + L \tag{2}$$

with the following initial and boundary conditions:

$$V_\theta = 0 \text{ when } t = 0,\ r \geqslant a; \tag{3}$$

$$V_\theta = a\Omega \text{ when } r = a; \tag{4}$$

$$V_\theta \to 0 \text{ when } r \to \infty, \tag{5}$$

where V_θ is the θ component of vector V; I is the moment of inertia of unit length of the cylinder; Ω is the angular velocity of the cylinder; a is the radius of the cylinder; ν is the kinematic viscosity; $L = 2\pi\mu a^2(\partial V_\theta/\partial r|_{r=a} - \Omega)$ is the moment of viscous forces acting on unit length of the cylinder due to the fluid; $\mu = \rho\nu$; ρ is the density of the fluid.

To solve the posed problem we use an operational method. Converting to images in (1), (2), (4), and (5), we obtain

$$\frac{\partial^2 V_\theta^*}{\partial r^2} + \frac{1}{r}\frac{\partial V_\theta^*}{\partial r} - \left(\frac{1}{r^2} + \frac{p}{\nu}\right)V_\theta^* = 0; \tag{6}$$

$$Ip\Omega^* = M^* + L^*; \tag{7}$$

$$V_\theta^* = a\Omega^* \quad \text{when } r = a; \tag{8}$$

$$V_\theta^* \to 0 \quad \text{when } r \to \infty, \tag{9}$$

where

$$V_\theta^* = \int_0^\infty e^{-pt} V_\theta dt; \quad \Omega^* = \int_0^\infty e^{-pt}\Omega dt;$$

$$M^* = \frac{M}{p}, \quad L^* = 2\pi\mu a^2\left(\left.\frac{\partial V_\theta^*}{\partial r}\right|_{r=a} - \Omega^*\right); \tag{10}$$

p is a complex variable.

The solution of Eq. (6) satisfying conditions (8), (9) has the form

$$V_\theta^* = a\Omega^* \frac{K_1\left(r\frac{p^{1/2}}{\nu^{1/2}}\right)}{K_1\left(a\frac{p^{1/2}}{\nu^{1/2}}\right)}, \tag{11}$$

where K_1 is a MacDonald function. Using (7), (10), (11), we obtain

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